Solutions to Practice Midterm Exam

1. Weight of the oil in the left arm is equilibrated by the solid cylinder and the oil in the right arm:

\[ W_{\text{left}} = W_{\text{right}} \]

\[ \rho g (\text{Volume})_{\text{left}} = \rho g (\text{Volume})_{\text{right}} + W_{\text{cylinder}} \]

\[ \rho g (L + h)A = \rho ghA + W_{\text{cylinder}} \]

Solving this equation for \( W_{\text{cylinder}} \), we find that

\[ W_{\text{cylinder}} = \rho g (L + h)A - \rho ghA = \rho g LA \]

Answer: A

2. In air:

\[ mg = 30 \text{ N} \]

In water:

\[ mg - \rho_{\text{water}} g V = 20 \text{ N} \]

In the liquid:

\[ mg - \rho_{\text{liquid}} g V = 24 \text{ N} \]

Substituting \( mg = 30 \text{ N} \) in the second and third equation, we come to

\[ (30 \text{ N}) - \rho_{\text{water}} g V = 20 \text{ N} \]

\[ (30 \text{ N}) - \rho_{\text{liquid}} g V = 24 \text{ N} \]

It follows that

\[ \rho_{\text{liquid}} g V = 6 \text{ N} \]

\[ \rho_{\text{water}} g V = 10 \text{ N} \]

Dividing the two equations by one another, we come to \( (\rho_{\text{liquid}})/(\rho_{\text{water}}) = 6/10 \). Therefore, \( \rho_{\text{liquid}} = (6/10) \rho_{\text{water}} \)

Answer: B

3. Apply Eq. 15-17 on p. 337 of the Text, \( F_i = MgA/A_o \). If \( A_i = \pi R_i^2 \) and \( A_o = \pi R_o^2 \), then \( A/A_o = (\pi R_i^2)/(\pi R_o^2) = (R/R_o)^2 = (2R/2R_o)^2 = (D/D_o)^2 \), where \( D_i = 0.01 \text{ m} \) is the “input” diameter and \( D_o = 0.10 \text{ m} \) is the “output” diameter. This gives \( A/A_o = [(0.01 \text{ m})/(0.10 \text{ m})]^2 = 0.1^2 = 0.01 \). Plugging \( F_i = 250 \text{ N} \) and \( g = 9.8 \text{ m/s}^2 \), we come to \( 250 \text{ N} = M(0.01)(9.8 \text{ m/s}^2) \). Solving this equation for \( M \), we find: \( M = 2551 \text{ kg} \approx 2550 \text{ kg} \).

Answer: D

4. Mercury is an incompressible fluid, therefore, its volume flow rate is constant, \( Av = \text{const} \) (see Eq. 16-3 and Eq. 16-4 on p. 353). In this case we can apply Eq. 16-3: \( A_1 v_1 = A_2 v_2 \).
Plugging $A_1 = 12.0 \text{ cm}^2$, $A_2 = 6.0 \text{ cm}^2$, and $v_2 = 8.0 \text{ m/s}$, we come to the following equation:

$$(12.0 \text{ cm}^2)v_\Lambda = (6.0 \text{ cm}^2)(8.0 \text{ m/s}).$$

Solving for $v_\Lambda$, we find $v_\Lambda = 4.0 \text{ m/s}$. Answer: B

5. Apply the Bernoulli’s equation:

$$p_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2.$$ 

In this case $p_1 = p_2$, therefore

$$\rho gh_1 + \frac{1}{2}\rho v_1^2 = \rho gh_2 + \frac{1}{2}\rho v_2^2$$

It follows that

$$2gh_1 + v_1^2 = 2gh_2 + v_2^2$$

$$v_2^2 = v_1^2 + 2g(h_1 - h_2) = (5 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(0.50 \text{ m}) = 34.8 \text{ (m/s)}^2.$$

Therefore, $v_2 = 5.9 \text{ m/s}$. Equation of continuity gives $v_1A_1 = v_2A_2$. It follows that $A_2 = (v_1/v_2)A_1 = [(5.0 \text{ m/s})/(5.9 \text{ m/s})](3.0 \times 10^{-5} \text{ m}^2) = 2.5 \times 10^{-5} \text{ m}^2$. Answer: C

6. 1 kg of He = 1000 g of He = 1000/4 moles = 250 moles. In 1 mole there are $N_A = 6.02 \times 10^{24}$ atoms of He. Therefore, in 250 moles there are $250 \times 6.02 \times 10^{23} = 1.5 \times 10^{26}$ atoms of He. Each He atom has two electrons, their electric charge being $2 \times 1.6 \times 10^{-19} \text{ C} = 3.2 \times 10^{-19} \text{ C}$. This results in a total of $(1.5 \times 10^{26})(3.2 \times 10^{-19} \text{ C}) = 4.8 \times 10^7 \text{ C}$ in 1 g of He. Answer: C

7. $F = \frac{k(Q - q)q}{(1 \text{ m})^2} = k(Q - q)q$. Therefore the function $F(q)$ is a maximum under condition that $\frac{dF}{dq} = 0$. Therefore, $d[k(Qq - q^2)]/dq = kd(Qq - q^2)/dq = k(Q - 2q) = 0$. It follows that $q = Q/2$.

Answer: C

8. $F = kQq/r^2$.

$$r^2 = a^2 + d^2,$$ therefore $F = kQq/(a^2 + d^2)$.

$F_x = F \cos \theta$,

$$\cos \theta = dl/r = dl\sqrt{(a^2 + d^2)}$$

$$F_x = [kQq/(a^2 + d^2)] \left[dl\sqrt{(a^2 + d^2)}\right]$$

$$= (kQq/\alpha^2)[u(1 + u^2)^{3/2}].$$

(\text{(*)})

Here the notation $u = d/\alpha$ was introduced. As it follows from the latter equation (\text{(*)}), $F_x$ takes the greatest value when the factor $f(u) = u(1 + u^2)^{3/2}$ is
at maximum. To find the maximum value of the function \( f(u) \), apply calculus. At maximum the derivative of \( f(u) \) equals zero, \( df/du = 0 \). This results in the following equation:

\[
(1 - 2u^2)(1 + u^2)^{-5/2} = 0.
\]

Solving this equation for \( u \), we find \( u = \pm 1/\sqrt{2} \). This gives \( d\alpha = \pm \alpha/\sqrt{2} \).

Answer: E

9. At \( r_1 = 2 \text{ m} \) \( E_1 = E \). At \( r_2 = 1 \text{ m} \) \( E_2 = x \). From the other hand, \( E_1 = kq/(r_1)^2 \) and \( E_2 = kq/x^2 \). Therefore, \( E_2/E_1 = [kq/(r_2)^2]/[kq/(r_1)^2] = (r_1)^2/(r_2)^2 = (r_1/r_2)^2 = [(2 \text{ m})/(1 \text{ m})]^2 = 4 \). Therefore, \( x/E = 4 \), and \( x = 4E \).

Answer: C

10. According to the principle of superposition for electric field (Eq. 26-7 on p. 590 of the Text), the vector of electric field at P is a resultant of four contributions, from each of the four point sources at the corners of the square. Two contributions cancel each other, the ones originating from the two opposite corners with +4 \( \mu \text{C} \) at each. These two vectors are equal in magnitude and have opposite direction. The other two vectors, \( \textbf{E}_3 \) due to the charge -3 \( \mu \text{C} \) at the left bottom corner, and the vector \( \textbf{E}_4 \) due to the charge +1 \( \mu \text{C} \) at the right top corner, have same direction, to the left bottom corner, and do not cancel. The corresponding magnitude of the resultant vector, \( E = |\textbf{E}_3| + |\textbf{E}_4| = kq/a^2 + 3kq/a^2 \), where \( q = 1 \mu \text{C} \), \( 3q = 3 \mu \text{C} \), and \( a = (2 \text{ m})/\sqrt{2} = 1.41 \text{ m} \) is distance from the point P at the center of the square to its corner. The above equation gives \( E = (kq/a^2)(1 + 3) = 4 \) \( kq/a^2 = 4(9 \times 10^9)(1 \times 10^6 \text{ C})/(1.41 \text{ m})^2 = 1.8 \times 10^4 \text{ N/C} \).

Answer: D

11. \( U = -pE \cos \theta \). The initial value \( \theta_i = 0^\circ \), \( \cos 0^\circ = 1 \), therefore \( U_i = -pE \).

Case 1: The final value \( \theta_f = 45^\circ \), \( \cos 45^\circ = 0.707 \), therefore \( U_f = -0.707pE \). For the change in the potential energy we find \( \Delta U = U_f - U_i = -0.707pE - (-pE) = 0.293 \text{ pE} \)

Case 2: The final value \( \theta_f = 90^\circ \), \( \cos 90^\circ = 0 \), therefore \( U_f = 0 \). For the change in the potential energy we find \( \Delta U = U_f - U_i = 0 - (-pE) = pE \)

Case 3: The final value \( \theta_f = 135^\circ \), \( \cos 135^\circ = -0.707 \), therefore \( U_f = 0.707pE \). For the change in the potential energy we find \( \Delta U = U_f - U_i = 0.707pE - (-pE) = 1.707 \text{ pE} \)

Case 4: The final value \( \theta_f = 180^\circ \), \( \cos 180^\circ = -1 \), therefore \( U_f = pE \). For the change in the potential energy we find \( \Delta U = U_f - U_i = pE - (-pE) = 2pE \)

Ranked, least to greatest, the change in the potential energy is: 1, 2 3, 4.

Answer: A

12. Joining 8 such cubes together, we get a bigger cube with the point charge \( Q = 5.0 \times 10^6 \text{ C} \) at its center. Then for one small cube,
\[ \Phi = \frac{1}{8} \Phi_\cdot = \frac{1}{8}(Q/\varepsilon_0) = \frac{1}{8}4\pi kQ = \frac{1}{2}\pi kQ \]
\[ = 0.5(3.14)(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.0 \times 10^{-6} \text{ C}) = 7.06 \times 10^4 \text{ N} \cdot \text{m}^2/\text{C} \]

Answer: B

13. Apply Eq. 26-16 on p. 593 of the Text. Plugging \( \lambda L = -1.0 \times 10^5 \text{ C}, y = 7.5 \times 10^{-3} \text{ m}, \) and \( L = 0.50 \text{ m}, \) we find: \( E = (9 \times 10^9)(1.0 \times 10^5 \text{ C})/(7.5 \times 10^{-3} \text{ m})\sqrt{[(7.5 \times 10^{-3} \text{ m})^2 + (0.25 \text{ m})^2]} = 4.8 \times 10^7 \text{ N/C}. \)

Answer: E

14. According to Gauss’ Law, \( \Phi_\circ = \frac{Q_{\text{inside}}}{\varepsilon_0} = q/\varepsilon_0. \) From another hand, due to spherical symmetry, \( \Phi_\circ = (4\pi r^2)E. \) Comparing these two equations, we find that \( E = q/(4\pi r^2 \varepsilon_0). \)

Answer: C

15. All electric charge of the hollow conductor is on its outer surface. Therefore, the inner surface is uncharged. No charge can be transferred to the metal ball.

Answer: C

16. See Fig. 2 above. \( U_{12} = kq^2/(5 \text{ cm}) \)

\[ q_1 = q \quad q_2 = q \quad q_3 = -q \]

\[ 0 \quad 5 \quad x \]

Figure 2

\[ U_{13} = -kq^2/|x| \]
\[ U_{23} = -kq^2/(|x - 5| \text{ cm}). \]

Therefore, \( U = U_{12} + U_{13} + U_{23} = kq^2[(1/5) - (1/|x|) - 1/(|x - 5|)]. \)

At infinite separation \( U = 0, \) therefore, the condition to satisfy is \( (1/5) - (1/|x|) - 1/(|x - 5|) = 0, \) or \( |x|^{-1} + |x - 5|^{-1} = 0.2. \) This equation has two solutions: \( x = -8.09017 \text{ cm} \approx 8.1 \text{ cm} \) and \( x = 13.09017 \text{ cm} \approx 13.1 \text{ cm}. \) Actually, both positions are at the same distance \( 10.59017 \text{ cm} \approx 10.6 \text{ cm} \) from the midpoint between the equal charges \( q. \) Of the two answers, \( 13 \text{ cm} \) is present in the list of possible answers. Therefore, Answer: A

17. After the two spheres are connected by a conducting wire, the electric charge distributes over the two objects so that the electric potential of the two spheres becomes equal, \( V_1 = V_2. \) Electric charge of the big sphere takes an unknown value \( Q \) and the one of the small sphere becomes \( q, \) also unknown. Due to conservation of electric charge, the total, \( Q + q, \) has to remain equal the initial value, \( Q_0 = 6 \times 10^8 \text{ C}. \) Besides, from the condition that \( V_1 = V_2 \) we find that \( kQ/(2R) = kq/R. \) It follows that \( Q = 2q. \) Solving this equation together with the above mentioned equation \( Q + q = Q_0, \) we find that \( 3q = Q_0, \) therefore \( q = Q_0/3 = (1/3)(6 \times 10^8 \text{ C}) = 2 \times 10^8 \text{ C} \) and \( Q = 2q = 2(2 \times 10^8 \text{ C}) = 4 \times 10^8 \text{ C}. \)

Answer: B
18. Relative to the potential far away, electric potential of a charged conducting sphere is \( V = Q/(4\pi \varepsilon_0 R) \). It follows that \( Q = 4\pi \varepsilon_0 RV \). Therefore, surface charge density is \( Q/A = Q/(4\pi R^2) = \varepsilon_0 V/R = (-100 \text{ V})(8.85 \times 10^{-12} \text{ C}^2/\text{m}^2 \cdot \text{N})/(5 \times 10^{-2} \text{ m}) = -1.77 \times 10^{-8} \text{ C/m}^2 \).

Answer: E

19. Because \( \mathbf{D} = x \mathbf{i} + y \mathbf{j} \) is a radial vector that is in the \( xy \) plane, the given electric field \( \mathbf{E} = C\rho \) is points in radial directions. Equipotential surfaces are perpendicular to the electric field lines, therefore, they are concentric cylinders with axes along the \( z \) axis.

Answer: A

20. According to definition of electric potential, it is equal to work required to bring a 1.0 C charge from infinity to a given point in the field. Therefore, the question in this problem can be reformulated as the request to find potential of the dipole at the point \( P \) on the perpendicular to its midpoint. We can apply Eq. 28-21 on p. 643 of the Text. At the point \( P \) on the perpendicular to its midpoint, the two distances, \( r_1 \) and \( r_2 \), in Eq. 28-21, are equal to one another. Therefore, the two terms in Eq. 28-21 cancel each other, and the resultant potential equals zero.

Answer: A