Data, Measurement, and Units Exercise *Name* ________________________________

(adapted from the oceanography lab manuals by RE Johnson and by HV Thurman & SM Savin)

Objectives:

* Define and relate the terms unit, measurement, and data.
* Measure height, width, and length of an object in both English and metric systems, calculating the object's volume, as well as the percent deviation of measurements.
* Define the term conversion factor and be able to give examples.
* Perform simple and multiple unit conversions, both within the metric system and between the English and metric systems.
* State and apply the rules for identifying the significant figures in a number.
* Round off calculations to the correct number of significant figures.
* State the rules for scientific notation and define the term exponent.
* Translate between regular notation and scientific notation.
* Perform, without a calculator, operations of addition, subtraction, multiplication, and division using scientific notation.
* Solve equations for a given variable and attain a final numerical value.
* Apply these techniques to examples relevant to geology

Introduction:

All physical quantities can be fundamentally expressed in terms of units of length, mass, and time. A unit, such as an inch, pound, or second, is an amount or quantity that serves as a standard against which other quantities are compared or measured. An accompanying numerical value relates the magnitude, or size, of any measurement to that standard. Thus, a measurement has two essential parts: a number and a unit. Examples of measurements are 12 feet, 16 pounds, and 60 minutes.

It is important to remember that the expression of any measurement in terms of only a numerical value is incomplete. For example, the number 8.5 alone does not express the width of this page. The numerical value, 8.5, and the unit of measurement, inches, must both be specified.

A collection or series of measurements is referred to as data. Scientific data are typically measured and recorded using the metric system of units because of the simplicity of converting among units. However, because the English (British) system of units is still prevalent in everyday life in the United States, the scientist must be familiar with the relationships between the two systems and be able to easily convert, or switch, units when necessary.

The Metric System:

A working proficiency in using the metric system can be attained by simply learning three base units and a few prefixes:

Base unit	Quantity	Abbreviation
meter	length	m
gram	mass	g
liter	volume	L

Prefix	Definition	Abbreviation	In Scientific Notation
giga	1,000,000,000 times the base unit	G	10^9 (billions)
mega	1,000,000 times the base unit	M	10^6 (millions)
kilo	1000 times the base unit	k	10^3 (thousands)
centi	1/100 of the base unit	c	10^{-2} (one-hundredth)
milli	1/1000 of the base unit	m	10^{-3} (one-thousandth)
micro	1/1,000,000 of the base unit	μ	10^{-6} (one-millionth)
nano	1/1,000,000,000 of the base unit	n	10^{-9} (one-billionth)

By combining a prefix and a base unit, a new unit that has a known relationship to the base unit is formed. For example:

Prefix	+	Base Unit	→	New Base Unit
kilo (k)	+	meter (m)	→	1000 m = 1 km
centi (c)	+	meter (m)	→	1/100 m = 0.01 m = 1 cm

Converting Units:

Often it is necessary to change the unit of a measurement to a different unit, of either the same or a different system. To convert a unit, the "conversion factor" must be known. A conversion factor is the relationship between the old unit and the new unit. For example, to convert from feet to inches, the conversion factor, 12 inches = 1 foot, is necessary. ***Table 1*** lists some commonly used conversion factors, both within the metric system and between the metric and English systems.

Consider units as algebraic quantities that may be multiplied or divided by one another. In converting units, we can take advantage of three facts:

1. Multiplying or dividing anything by one (1) leaves its value unchanged.
2. Any conversion factor, constructed as a ratio, will yield a value of one (1).
3. Units cancel, just as numbers do.

Therefore, to perform a simple conversion, for example: 18 yards = ? feet, find the appropriate conversion factor (***Table 1***) and set up a conversion bracket as shown:

$$18 \text{ yd} \quad \times \quad (\quad) \quad = \quad ?\text{ ft}$$

Next, insert the conversion factor (1 yd = 3 ft) into the bracket in such a way that you can multiply through, cancelling out the old units (yd) and retaining the new (ft).

$$18 \text{ yd} \quad \times \quad \left(\frac{3 \text{ ft}}{1 \text{ yd}}\right) \quad = \quad ?\text{ ft}$$

Now multiply the numbers and units, to find the answer.

$$18 \text{ yd} \quad \times \quad \left(\frac{3 \text{ ft}}{1 \text{ yd}}\right) \quad = \quad 54 \text{ ft}$$

In this example, we can see that the units of yards from the "18 yd" and the yards from the "1 yd" will cancel, leaving us with the units of feet. This is because yards in the numerator (top) cancel with yards in the denominator (bottom) when the two parts are multiplied together. Note that if we had mistakenly inserted the conversion factor in to the bracket *upside-down*, the units (yd) would not have cancelled out (we'd have been left with units of yd^2/ft!). The cancellation of like units serves as a check for the proper set up of any equation.

In a case where the conversion factor between two units is not known directly, a multiple-step conversion may be performed, using intermediate conversion factors.

For example: 20 km = ? mm

Conversion factors (from ***Table 1***): 1 km = 1000 m & 1 m = 1000 mm

$$20 \text{ km} \quad \times \quad \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) \quad \times \quad \left(\frac{1000 \text{ mm}}{1 \text{ m}} \right) \quad = \quad 20{,}000{,}000 \text{ mm}$$

Rounding off Numbers:

Frequently, it is necessary for scientists to round off numbers that result from various calculations performed on data. The decision of how and when it is appropriate to round off a number is not an arbitrary one. The digit following the specified place, if 5 or greater, is rounded up, and if the digit is less than 5, then it is rounded down. The following example illustrates how to round off the number 87392.634 to a variety of specified places:

100ths	87392.63
10ths	87392.6
Integer	87393
10's	87390
100's	87400
1000's	87000
10,000's	90000

Significant Figures:

To decide to what place we must round a calculated result, we must first determine the number of significant figures in each of our original measurements. This determination is based on the following criteria:

1. All non-zero digits are significant (i.e., 1,2,3,4,5,6,7,8,9)

2. Zeroes between other non-zero digits are significant (e.g., 10405)

3. Terminal zeroes to the right of the decimal are significant (e.g., 123.40, and 345.900)

4. Zeroes before the first non-zero digit and/or those that only show magnitude are *not* significant (e.g., 0.000576, and 900000)

When rounding off the result of a calculation, that result can be no more accurate than the ***least*** accurate measurement. There are essentially two rules to remember:

1. When adding and subtracting, the result may not have a digit beyond the first decimal position beyond which, for any measurement, nothing is recorded.

2.1 mm			
3.123 mm		9.725 km	
+ 1.01 mm		- 4.04 km	
6.233 mm → Rounds off to ***6.2 mm***		5.685 km → Rounds off to ***5.68 km***	

2. When multiplying and dividing, the result may not have more significant figures than the measurement with the least number of significant figures.

2.112 hr	73.28 km
x 4.3 km/hr	÷ 18.137 km/hr
9.0816 hr → Rounds to ***9.1 hr***	4.04036 hr → Rounds to ***4.040 hr***

Scientific Notation:

Very large and very small numbers are commonly encountered in science and are better expressed in scientific notation. Any number can be expressed in decimal form as a number between 1 and 10 multiplied by a power of ten (exponent). This exponent indicates in which direction and how many places the decimal must be moved to express the number in its expanded form.

For example:

$79{,}200 = 7.92 \times 10^{4}$ $\quad$ $0.000792 = 7.92 \times 10^{-4}$

Therefore, a scientific notation with a positive exponent represents a number greater than one (1), while a scientific notation with a negative exponent denotes a number less than one (1). Also note that the digits appearing in decimal form in the scientific notation are the significant numbers. Refer to ***Table 2*** for an expanded illustration of the powers of 10 covering 13 orders of magnitude.

1. When adding or subtracting using scientific notation, numbers must be converted to the same power of 10:

$(3 \times 10^{5}) + (2 \times 10^{4}) = (3 \times 10^{5}) + (0.2 \times 10^{5}) = 3.2 \times 10^{5}$

$(6 \times 10^{-4}) - (4 \times 10^{-5}) = (6 \times 10^{-4}) - (0.4 \times 10^{-4}) = 5.6 \times 10^{-4}$

2. When multiplying or dividing using scientific notation, the exponents are ***added*** (multiplication) or ***subtracted*** (division):

$(3 \times 10^{3}) \times (2 \times 10^{2}) = 6 \times 10^{(3+2)} = 6 \times 10^{5}$

$(9 \times 10^{8}) \div (3 \times 10^{5}) = 3 \times 10^{(8-5)} = 3 \times 10^{3}$

$(9 \times 10^{8}) \div (3 \times 10^{-3}) = 3 \times 10^{[8-(-3)]} = 3 \times 10^{11}$

Solving Equations:

Scientists perform various calculations to consolidate or clarify their data and to help them answer the questions they have posed. To perform the calculations required in a geology lab, follow these suggestions:

1. Choose the equation that relates the known values available to you and the unknown value you are asked to calculate.

2. Solve the equation for the unknown. To simplify the equation and isolate the unknown, you may perform any operation (addition, subtraction, multiplication, division) so long as it is done on ***both*** sides of the equal sign. Also recall that the order of the operations is: "Pretty Please My Dear Aunt Sally", where the letters stand for: Powers, Parentheses, Multiplication and Division, Addition and Subtraction.

3. Carefully insert the known values into their appropriate positions in the equation. Always include the units as well as the numerical values!

4. Report the calculated result in terms of a numerical value and appropriate units. Check through your equations to assure yourself that all units except the final ones cancel out. Do the units of your result pertain to the question asked? Does the numerical value seem ***reasonable***?

5. Round off your calculated result to the appropriate number of significant figures.

NOTE: When performing calculations, you should solve the equation for the unknown variable ***before*** plugging in any numbers.

<u>Examples:</u>

A. Solve the equation $d = vt$ for "v".
Solve the equation $d = vt$ for "t".

This is "Distance equals Speed (Velocity) times Time", an equation that's used frequently in geology! For example, how fast does a river flow? How fast do tectonic plates move?

For the first calculation, you are asked to isolate "v" on one side of the equation and everything else on the other side of the equation. (Remember to perform the same operation on both sides of the equal sign in each step!) Remember that "vt" means "v *times* t". So you need to *divide* both sides by "t" in order to get "v" alone.

Solving the equation $d = vt$ for "v":

$$\frac{d}{t} = \frac{vt}{t} \rightarrow \frac{d}{t} = v$$ (You can use this to find speed if you're given distance and time.)

Solving the equation $d = vt$ for "t":

$$\frac{d}{v} = \frac{vt}{v} \rightarrow \frac{d}{v} = t$$ (You can use this to find time if you're given distance and speed.)

B. Solve the equation $x = y + vt + \frac{1}{2}at^2$ for "a".

Take this stepwise, finally isolating "a" on one side of the equation and everything else on the other side of the equation. (Remember to perform the same operation on both sides of the equal sign in each step!) The simplest thing to do is start by subtracting terms from both sides to get the whole term with "a" in it alone on one side of the equation, then perform the steps required to get just "a" alone.

Step one:

$$\begin{array}{lcll} x & = & y + & vt + \frac{1}{2}at^2 \\ \underline{\boldsymbol{- y}} & & \underline{\boldsymbol{- y}} & \\ x - y & = & & vt + \frac{1}{2}at^2 \end{array}$$

This can also be written as: $x \boldsymbol{- y} = y + vt + \frac{1}{2}at^2 \boldsymbol{- y}$

Or as: $(x) \boldsymbol{- y} = (y + vt + \frac{1}{2}at^2) \boldsymbol{- y}$

Step two:

$$\begin{array}{lcll} x - y & = & vt + & \frac{1}{2}at^2 \\ \underline{\boldsymbol{- vt}} & & \underline{\boldsymbol{- vt}} & \\ x - y - vt & = & & \frac{1}{2}at^2 \end{array}$$

Remember that "vt" means "v times t". This means they can "travel together" in this step. Now that you have $\frac{1}{2}at^2$ alone on one side of the equation, you need to get just the "a" by itself.

Step three: $x - y - vt = \frac{1}{2}at^2$

Let's get rid of the $\frac{1}{2}$ at the front of $\frac{1}{2}at^2$. Recall that two halves equal one (i.e., 2 times $\frac{1}{2}$ = 1), so this involves multiplying both sides of the equation by 2:

$$2 (x - y - vt) = 2 (\tfrac{1}{2}at^2)$$

$$2 (x - y - vt) = at^2$$

Step four: $2 (x - y - vt) = at^2$

Now the only thing left to do to get "a" alone, is to divide both sides by t^2:

$$\frac{2 (x - y - vt)}{t^2} = \frac{at^2}{t^2}$$

So…

$$\frac{2 (x - y - vt)}{t^2} = a$$ *(whew!)*

Table 1 - Conversion Factors

Units of Length

1 kilometer (km)	=	1000 meters (m)				
1 meter (m)	=	100 centimeters (cm)	=	1000 millimeters (mm)		
1 foot (ft)	=	12 inches (in)				
1 yard (yd)	=	3 feet (ft)				
1 mile (mi)	=	5280 feet (ft)	=	1.6 kilometers		
1 meter (m)	=	3.28 feet (ft)		(x 1yd/3ft)	=	1.093 yard (yd)
1 yard (yd)	=	0.9146 meter (m)		(x 100cm/1m)	=	91.46 centimeters (cm)
1 centimeter (cm)	=	0.3937 inch (in)				
1 inch (in)	=	2.54 centimeters (cm)				

Units of Volume

1 liter (L)	=	1000 milliliters (mL)	=	1000 cubic centimeters (cm^3)	=	1.0567 quart (qt)
1 milliliter (mL)	=	1 cm^3				
1 cubic meter (m^3)	=	1000 liters (L)				

Units of Mass

1 kilogram (kg)	=	1000 grams (g)	=	2.205 pounds (lb)
1 gram (g)	=	1000 milligrams (mg)	=	0.035 ounce (oz)
1 metric ton	=	1,000,000 grams (g)	=	2205 pounds (lb)
1 English ton	=	2000 pounds (lb)		
1 pound (lb)	=	454.6 grams (g)		

Units of Temperature

Boiling point of pure water at 1 atm of pressure	=	100° Centigrade (°C)	=	212° Fahrenheit (°F)
Freezing point of pure water at 1 atm of pressure	=	0° Centigrade (°C)	=	32° Fahrenheit (°F)

°F	=	(°C x 1.8) + 32	=	(9°C + 160) / 5
°C	=	0.56 (°F - 32)	=	(5°F - 160) / 9

Table 2 - Powers of 10 from 10^{-6} to 10^{6}

1×10^{-6}	=	0.000001	=	1 with decimal point moved **6** places to the *left*
1×10^{-5}	=	0.00001	=	1 with decimal point moved **5** places to the *left*
1×10^{-4}	=	0.0001	=	1 with decimal point moved **4** places to the *left*
1×10^{-3}	=	0.001	=	1 with decimal point moved **3** places to the *left*
1×10^{-2}	=	0.01	=	1 with decimal point moved **2** places to the *left*
1×10^{-1}	=	0.1	=	1 with decimal point moved **1** place to the *left*
1×10^{0}	=	1	=	1 with decimal point moved **0** places to the left or right
1×10^{1}	=	10	=	1 with decimal point moved **1** place to the *right*
1×10^{2}	=	100	=	1 with decimal point moved **2** places to the *right*
1×10^{3}	=	1000	=	1 with decimal point moved **3** places to the *right*
1×10^{4}	=	10,000	=	1 with decimal point moved **4** places to the *right*
1×10^{5}	=	100,000	=	1 with decimal point moved **5** places to the *right*
1×10^{6}	=	1,000,000	=	1 with decimal point moved **6** places to the *right*

Exercise: Data, Measurements, and Units **OPTIONAL, FOR PRACTICE**

Your name: ____

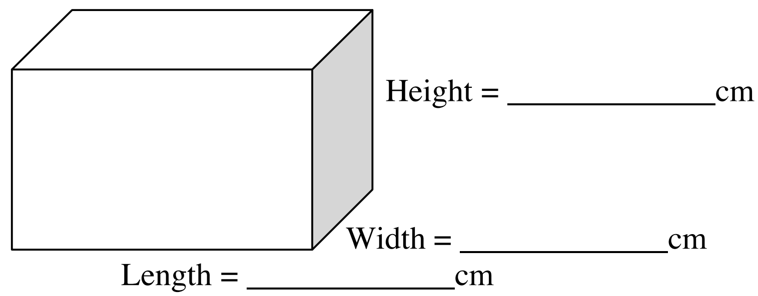
For this exercise and all to follow, use pencil only. Show all work when performing calculations. Show each step, even if you can do it in your head. Remember to always include units, in every part of every step. Write very neatly - If I can't read it, I can't grade it.

1. Measure the height, width, and length of the outside of a box, in centimeters, to the nearest tenth of a centimeter (that is, to one decimal place). With these measurements, calculate the volume of the box in both cubic centimeters (cm^3) and in liters (L).

Volume of a box = Length x Width x Height

Volume: ___________ cm^3

Volume: ___________ L



2. If that box were to be filled with fresh water, how much would it weigh in kilograms (kg)? In pounds (lb)? Assume that ***1 gram (g) = 1 cm^3*** for the density of fresh water at 25ºC (room temperature).

Mass: ___________ kg   Mass: ___________ lb

3. Four students per group are to measure the length of an object provided by the instructor to the nearest tenth of a mm. Each student is to determine the length without communicating with the others in the group, until everyone has completed their measurements.

#1: ____________mm #2: ____________mm #3: ____________mm #4: ____________mm

a. What is the average value for the length?: ____________mm

Average value = (Sum of individual values) ÷ (Number of individual values being averaged)

b. What is the percent deviation of each measurement from the average value, given the following equation?:

$$\textbf{\% Deviation} = \left(\frac{\text{measured - average}}{\text{average}}\right) \textbf{x 100\%}$$

#1: ____________% #2: ____________% #3: ____________% #4: ____________%

4. Convert the following:

a. 8 km = ? cm

b. 5 meters = ? feet

c. 25ºC = ? ºF

5. **a.** Determine the number of significant figures in the measurements below:

730,000 years has ________________________ significant figures.

0.00305 grams has ________________________ significant figures.

506.03 meters has ________________________ significant figures.

b. Express these numbers in scientific notation:

2050 yr is ________________________ in scientific notation.

0.116 mi is ________________________ in scientific notation.

9.053 kg is ________________________ in scientific notation.

6. Given the equation $2X = \frac{6}{Y} + 4Z$, solve for Y.

7. We generally think of the oceans as being deep, but it is useful to have some feeling for the depth of the ocean relative to its horizontal dimensions. The average depth of the ocean is about 3.8 km. We can't really define an average width, but depending on where we measure across the oceans, we typically get distances of 5000 km to 10,000 km. Let's pick a width of 7600 km to keep the arithmetic simple.

a. Start by calculating the depth-to-width ratio (that is, depth ÷ width). Notice that as both values are listed in km, the units cancel and we're left with a unitless ratio.

b. Now imagine that we are trying to build a scale model of the ocean with the same depth-to-width ratio as the real ocean. Let's build our model in a pan that is 20 inches across. How deep should the water be in our model ocean?

8. **Let's calculate the average density of our planet.** The mass of the Earth can be estimated quite accurately from the way objects are attracted to it by gravity. I'll spare you the computations, but we will use a figure for the mass of the Earth of 5.98×10^{27} g. The radius of the Earth is approximately 6370 km.

a. What is the radius of the Earth in centimeters?: ________________cm
(Hint: Convert 6370 km to ? cm. Convert to scientific notation, to eliminate those pesky zeroes.)

b. What is the volume of the Earth, in cubic centimeters? (You can use the value of 6370 km for the radius of the Earth, but remember to convert that value to cm in order to calculate the volume of the Earth in cm^3.) We're assuming here that Earth is a perfect sphere. The volume of a sphere is:

$$\textbf{\textit{Volume of a sphere}} = \frac{4}{3}\pi r^3$$

(where r = radius, and π = ratio of circumference to diameter of a sphere and $\pi \approx 3.1416927$)

The volume of Earth is: ________________cm^3

c. Now let's calculate the average density of our planet, in grams per cubic centimeter (g/cm^3). The density of a substance is how "heavy" the material is - that is, how much a specified volume of the material would weigh. Mathematically, the equation is:

Density = Mass ÷ Volume

(Remember, you were given the mass in grams in part a, and calculated the volume in cm^3 in part b.)

The average density of Earth is: ________________g/cm^3

The average density of the rocks at the surface of the Earth is about 2.7 g/cm^3. So if your answer to part c was correct, you realize that the average density of our planet is considerably greater than the average density of the rocks at its surface. This tells us that the density of the material in the interior of the Earth is considerably greater than the density of material at the surface. We are unable to make direct observations or take samples of the deep interior of the Earth, so our information about what's down there must come from indirect evidence. Among the lines of evidence that geophysicists use to draw conclusions about the nature of the Earth's interior is the density of the Earth, and how the density varies with depth below the surface. In other words, calculations like those you just made tell us very important things about the parts of the Earth we cannot examine directly.

9. **<u>In this question we will consider the origin of water in Earth's hydrosphere.</u>** There is good reason to believe that most of the water in the hydrosphere escaped to the surface of the planet fairly early in Earth's history. This process of transport of water from Earth's mantle to the surface is called <u>outgassing</u>. We can see water coming from the interior to the surface today, such as in the form of hot springs, geysers, and volcanic emissions. Almost all of that water, however, originated as rainwater that percolated downward into the crust and reacted with hot rocks before returning to the surface. But a small amount of the water coming to the surface of the Earth today (and undoubtedly much more when the Earth was a young planet!) is <u>juvenile water</u>, or water that is reaching the surface for the first time. Geologists are generally in agreement that the Earth formed about 4.5 billion years ago as the result of the agglomeration of meteorites (essentially). Most meteorites consist either of an alloy of iron (Fe) and nickel (Ni), or of aluminosilicate minerals (rich in aluminum [Al], silicon [Si], and oxygen [O]) similar to those that make up the Earth's mantle and parts of the crust. You will estimate the amount of water that might have existed in the mantle of the Earth early in its history (once it differentiated into layers). You will then compare that amount with the amount of water in the oceans. Finally you will conclude whether or not the water of the oceans could have originated entirely by outgassing of Earth's interior.

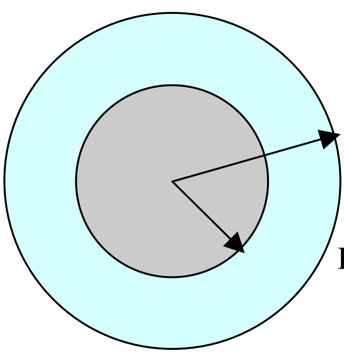
a. First, calculate the <u>volume</u> of the Earth's mantle, in cubic centimeters. A cross-section of the planet is shown below. For this question we'll ignore the Earth's crust, which is very thin and thus contributes very little to the total mass of the planet. Remember that the volume of a sphere is given by:

Volume of a sphere $= \frac{4}{3}\pi r^3$

(where r = radius, and π = ratio of circumference to diameter of a sphere and $\pi \approx 3.1415927$)

Use these data to calculate the volume of the mantle. Remember that the Earth and the core are spheres, but the mantle is *not* a sphere! The mantle is the part of the Earth that is not core, so the volume of the mantle is:

Volume of mantle = (Volume of Earth) – (Volume of core) or $V_{mantle} = V_{Earth} - V_{core}$



Radius of Earth = 6370 km

Radius of core = 3486 km

Volume of Earth: _________________ km^3

Volume of Earth: _________________ cm^3

Volume of core: _________________ km^3

Volume of core: _________________ cm^3

Volume of mantle: _________________ km^3

Volume of mantle: _________________ cm^3

HINTS: If you do the calculation using radii expressed in km, the answer you get will be in km^3 (cubic kilometers). You have been asked to express the answer in cm^3 (cubic centimeters). ***1 km³ = 10¹⁵ cm³*** (see part d for the derivation). Start by calculating the volume of the Earth and the volume of the core, then subtract to get the volume of the mantle. I think it is easiest to convert from km^3 to cm^3 as the last step, but you can do it sooner in the calculations if you wish.

b. Now calculate the mass of the mantle, in grams. Assume the density of the mantle is 4.5 g/cm^3 (about 4.5 times the density of water). Remember that:

Density = Mass ÷ Volume which can be rearranged to ***Mass = Density x Volume***

For our calculations, the units to use are: ***Mass (g) = Density (g/cm³) x Volume (cm³)***
(Notice that the units of cm^3 cancel, leaving g.)

Mass of mantle: _________________g

c. Next, assume that the mantle is made up of material from stony meteorites. Stony meteorites contain about 0.5% water (half a percent, by weight). Calculate the mass of water in the mantle when it was formed from the meteorites. Express your answer in grams. (Take the mass of the mantle from part b, and figure out how much of that mass would have been water, given the proportions listed here.)

Mass of water in early-Earth mantle: _________________g

d. Now calculate the volume of the oceans, in cubic centimeters. To do this, take the surface area of the oceans to be 3.6×10^8 km^2. Take an average depth to be 3.8 km. Calculate the volume using the formula:

Volume (cm³) = Area (cm²) x Depth (cm) or ***Volume (km³) = Area (km²) x Depth (km)***

You may also want to use: 1 km = 10^5 cm → $1km^3 = (10^5 cm)^3 = 10^{15} cm^3$

Volume of seawater in the oceans: _________________cm^3

e. Next calculate the mass of pure water in the oceans, in grams. (The saltiness came later.) Assume that the density of seawater is 1.025 g/cm^3 and that seawater is 96.5% pure water (the rest is "salts", etc).

Mass of pure water in the oceans: _________________g

f. Finally, *compare* the mass of pure water in the oceans (part e) to the mass of water originally in the mantle (part c). Which is bigger? By how much? Could the water of the oceans have come entirely from outgassing of the mantle? Explain.

To calculate which is bigger, compare your answers from part e and part c of this question. To determine how many grams difference there is, subtract the two values. To calculate the percentage difference, use:

$$\textbf{\% difference} = \left(\frac{\text{difference between the values}}{\text{the larger value}}\right) \textbf{x 100\%}$$

or

$$\textbf{\% difference} = \left(\frac{\text{larger value - smaller value}}{\text{larger value}}\right) \textbf{x 100\%}$$

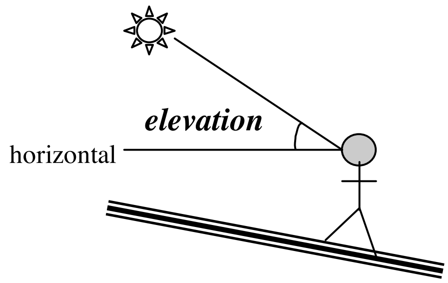
The mass of ______________________________________ is bigger.

It is bigger by _____________________ grams, which is a ____________ % difference.

Could the water of the oceans have come entirely from outgassing of the mantle? **Yes** or **No** .
Explain.

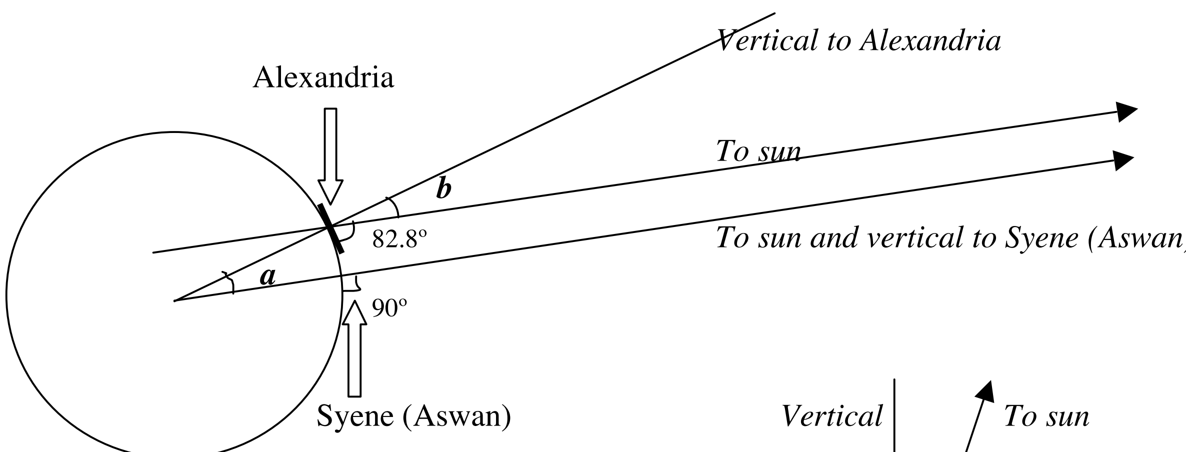
10. **<u>Here we will work through Eratosthenes' calculation of Earth's circumference (~230 B.C.).</u>**

The ***angle*** between the horizontal, an observer, and a celestial body is called the elevation of the body:

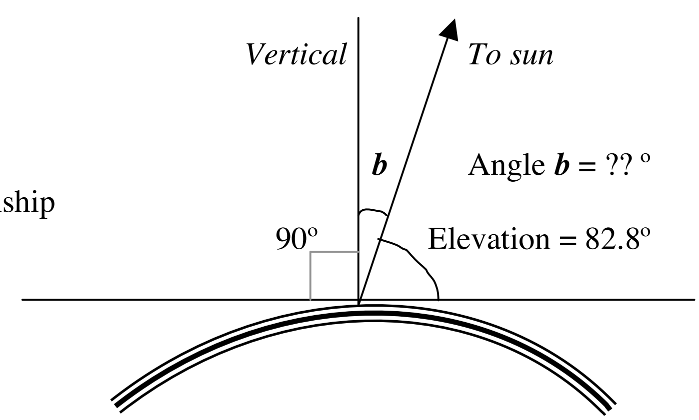


By measuring the elevation of the sun at two points along a north-south line on the surface of the Earth, and measuring the distance between those two points, it is possible to calculate the circumference of the Earth. Credit for the first measurement of this sort is often given to Eratosthenes (275-195 B.C.), a Greek who was the librarian of the famous library in Alexandria, Egypt. Eratosthenes noted that on one day each year the sun shone straight down a deep well in the city of Syene (now called Aswan). (It happened to be the summer solstice, but that is not essential to this measurement.) In other words, the elevation of the sun at noon in Syene on that day was 90º. At the same time in Alexandria, which is approximately 800 kilometers to the north of Syene, the elevation of the sun was 82.8º. (Noon is specified because at that time, shadows are aligned in a north-south direction.)

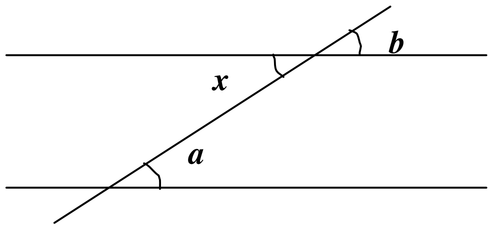
Below: On the day Eratosthenes made his measurements, at noon, the sun was directly overhead at Syene and was at an elevation of 82.8º at Alexandria. Because the sun is very far away from the Earth, the lines drawn to the sun from both Alexandria and Syene are parallel to each other.



At right: Close-up view of the relationship between the vertical and the direction to the sun at Alexandria.



As shown in the figure below, angle "***a***" (that is, the angle made by lines connecting the center of the Earth with Syene and Alexandria) is equal to angle "***b***" (that is, the angle made by a vertical line and a line pointing to the sun at Alexandria.)



angle ***a*** = angle ***x*** = angle ***b***

Above: When two parallel straight lines are cut by a third straight line, the interior angles formed are equal (that is, of the same size).

Using the preceding figures as a guide, and remembering that there are 360º in a circle (and 90º in a right angle), answer the following questions. ***Show all steps of your calculations.***:

a. How many degrees is angle "***a***", drawn from Alexandria to the center of the Earth to Syene? Explain your reasoning. (Hint: Be sure you look at all the diagrams!)

b. What fraction of 360º is the angle you just calculated?

c. Given that the distance from Alexandria to Syene is 800 km, use the answer above to compute the circumference of the Earth. Give your answer in kilometers. ***Voila!***

d. How close is your calculation of the circumference of the Earth to the actual circumference? Explain some of the reasons why the two numbers are different (list as many possible reasons as you can).