Data, Measurement, and Units Exercise *Name* ________ ANSWER KEY______

(adapted from the oceanography lab manuals by RE Johnson and by HV Thurman & SM Savin)

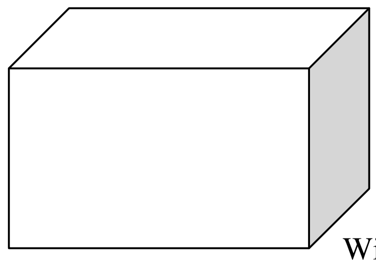
For this exercise and all to follow, use pencil only. Show all work when performing calculations. Show each step, even if you can do it in your head. Remember to always include units, in every part of every step. Write very neatly - If I can't read it, I can't grade it.

1. Measure the height, width, and length of the outside of a box, in centimeters, to the nearest tenth of a centimeter (that is, to one decimal place). With these measurements, calculate the volume of the box in both cubic centimeters (cm^3) and in liters (L).

Volume of a box = Length x Width x Height

Volume: ____*120,000*________cm^3

Volume: ____*120*_________L



Height = ____*40*_________cm

Width = ____*30*_________cm

Length = ____*100*_________cm

(I'm making up the dimensions.)

V = L x W x H = (100 cm) x (30 cm) x (40 cm) = 120,000 cm³

Recall that 1 liter (L) = 1000 milliliters (mL) = 1000 cubic centimeters (cm³)

120,000 cm³ x (1 L / 1000 cm³) = 120 L

2. If that box were to be filled with fresh water, how much would it weigh in kilograms (kg)? In pounds (lb)? Assume that ***1 gram (g) = 1 cm³*** for the density of fresh water at 25ºC (room temperature).

Mass: ___*120*_________ kg Mass: ____*264.6*_________lb

120,000 cm³ x (1 g / 1 cm³) = 120,000 g

Recall that 1 kg = 1000 g = 2.205 pounds (lb)

120,000 g x (1 kg / 1000 g) = 120 kg

120 kg x (2.205 lb / 1 kg) = 264.6 lb

3. Four students per group are to measure the length of an object provided by the instructor to the nearest tenth of a mm. Each student is to determine the length without communicating with the others in the group, until everyone has completed their measurements.

#1: _____________mm #2: _____________mm #3: _____________mm #4: _____________mm

a. What is the average value for the length?: _____________mm

Average value = (Sum of individual values) ÷ (Number of individual values being averaged)

b. What is the percent deviation of each measurement from the average value, given the following equation?:

$$\textbf{\% Deviation} = \left(\frac{\text{measured - average}}{\text{average}}\right) \times 100\%$$

#1: ____________*%* #2: ____________*%* #3: ____________*%* #4: ____________*%*

4. Convert the following:

a. 8 km = ? cm

Recall that 1 km = 1000 m and that 1 m = 100 cm

8 km x (1000 m / 1 km) x (100 cm / 1 m) = 800,000 cm

b. 5 meters = ? feet

Recall that 1 meter = 3.28 feet

5 meters x (3.28 feet / 1 meter) = 16.4 feet

c. 25°C = ? °F

Recall that °F = (°C x 1.8) + 32 = (9°C + 160) / 5

°F = (25 x 1.8) + 32 = 45 + 32 = 77°F

5. **a.** Determine the number of significant figures in the measurements below:

730,000 years has ____***2***____ significant figures.

0.00305 grams has ____***3***____ significant figures.

506.03 meters has ____***5***____ significant figures.

b. Express these numbers in scientific notation:

2050 yr is ____$\mathbf{2.050 \times 10^3}$____ in scientific notation.

0.116 mi is ____$\mathbf{1.16 \times 10^{-1}}$____ in scientific notation.

9.053 kg is ____$\mathbf{9.053 \times 10^0}$____ in scientific notation.

6. Given the equation $2X = \frac{6}{Y} + 4Z$, solve for Y.

2X = (6 / Y) + 4Z

2X – 4Z = (6 / Y) + 4Z – 4Z

2X – 4Z = (6 / Y)

Y * (2X – 4Z) = Y * (6 / Y)

Y * (2X – 4Z) = 6

[Y * (2X – 4Z)] / (2X – 4Z) = 6 / (2X – 4Z)

Y = 6 / (2X – 4Z)

7. We generally think of the oceans as being deep, but it is useful to have some feeling for the depth of the ocean relative to its horizontal dimensions. The average depth of the ocean is about 3.8 km. We can't really define an average width, but depending on where we measure across the oceans, we typically get distances of 5000 km to 10,000 km. Let's pick a width of 7600 km to keep the arithmetic simple.

a. Start by calculating the depth-to-width ratio (that is, depth ÷ width). Notice that as both values are listed in km, the units cancel and we're left with a unitless ratio.

D / W = 3.8 km / 7600 km = 0.0005

b. Now imagine that we are trying to build a scale model of the ocean with the same depth-to-width ratio as the real ocean. Let's build our model in a pan that is 20 inches across. How deep should the water be in our model ocean?

3.8 km / 7600 km = Q / 20 cm

0.0005 = Q / 20 cm

20 cm x 0.0005 = Q

Q = 0.1 cm deep

8. **Let's calculate the average density of our planet.** The mass of the Earth can be estimated quite accurately from the way objects are attracted to it by gravity. I'll spare you the computations, but we will use a figure for the mass of the Earth of 5.98×10^{27} g. The radius of the Earth is approximately 6370 km.

a. What is the radius of the Earth in centimeters?: _____***6.37×10^{8}***___________cm
(Hint: Convert 6370 km to ? cm. Convert to scientific notation, to eliminate those pesky zeroes.)

Recall that 1 km = 1000 m and 1 m = 100 cm

6370 km x (1000 m / 1 km) x (100 cm / 1 m) = 637,000,000 cm = 6.37×10^{8} cm

b. What is the volume of the Earth, in cubic centimeters? (You can use the value of 6370 km for the radius of the Earth, but remember to convert that value to cm in order to calculate the volume of the Earth in cm^3.) We're assuming here that Earth is a perfect sphere. The volume of a sphere is:

$$\textit{Volume of a sphere} = \frac{4}{3}\pi r^3$$

(where r = radius, and π = ratio of circumference to diameter of a sphere and π ≈ 3.1416927)

The volume of Earth is: ______*1.08×10^{27}*__________cm^3

$V = (4/3) \times \pi \times (6.37 \times 10^8\ cm)^3$ ← Notice that the "cubed" goes outside the parentheses

$V = (4/3) \times \pi \times (6.37)^3 \times (10^8)^3 \times (cm)^3$

$V = (4/3) \times \pi \times (258.47) \times 10^{24}\ cm^3$

$V = \boxed{1.08 \times 10^{27}\ cm^3}$

c. Now let's calculate the average density of our planet, in grams per cubic centimeter (g/cm^3). The density of a substance is how "heavy" the material is - that is, how much a specified volume of the material would weigh. Mathematically, the equation is:

Density = Mass ÷ Volume

(Remember, you were given the mass in grams in part a, and calculated the volume in cm^3 in part b.)

The average density of Earth is: ______*5.54*___________g/cm^3

$D = M / V$

$D = (5.98 \times 10^{27}\ g) / (1.08 \times 10^{27}\ cm^3)$

$D = \boxed{5.54\ g/cm^3}$

The average density of the rocks at the surface of the Earth is about 2.7 g/cm^3. So if your answer to part c was correct, you realize that the average density of our planet is considerably greater than the average density of the rocks at its surface. This tells us that the density of the material in the interior of the Earth is considerably greater than the density of material at the surface. We are unable to make direct observations or take samples of the deep interior of the Earth, so our information about what's down there must come from indirect evidence. Among the lines of evidence that geophysicists use to draw conclusions about the nature of the Earth's interior is the density of the Earth, and how the density varies with depth below the surface. In other words, calculations like those you just made tell us very important things about the parts of the Earth we cannot examine directly.

9. **In this question we will consider the origin of water in Earth's hydrosphere.** There is good reason to believe that most of the water in the hydrosphere escaped to the surface of the planet fairly early in Earth's history. This process of transport of water from Earth's mantle to the surface is called outgassing. We can see water coming from the interior to the surface today, such as in the form of hot springs,

geysers, and volcanic emissions. Almost all of that water, however, originated as rainwater that percolated downward into the crust and reacted with hot rocks before returning to the surface. But a small amount of the water coming to the surface of the Earth today (and undoubtedly much more when the Earth was a young planet!) is juvenile water, or water that is reaching the surface for the first time. Geologists are generally in agreement that the Earth formed about 4.5 billion years ago as the result of the agglomeration of meteorites (essentially). Most meteorites consist either of an alloy of iron (Fe) and nickel (Ni), or of aluminosilicate minerals (rich in aluminum [Al], silicon [Si], and oxygen [O]) similar to those that make up the Earth's mantle and parts of the crust. You will estimate the amount of water that might have existed in the mantle of the Earth early in its history (once it differentiated into layers). You will then compare that amount with the amount of water in the oceans. Finally you will conclude whether or not the water of the oceans could have originated entirely by outgassing of Earth's interior.

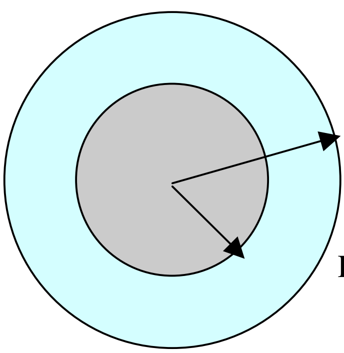
a. First, calculate the volume of the Earth's mantle, in cubic centimeters. A cross-section of the planet is shown below. For this question we'll ignore the Earth's crust, which is very thin and thus contributes very little to the total mass of the planet. Remember that the volume of a sphere is given by:

Volume of a sphere $= \frac{4}{3}\pi r^3$

(where r = radius, and π = ratio of circumference to diameter of a sphere and $\pi \approx 3.1415927$)

Use these data to calculate the volume of the mantle. Remember that the Earth and the core are spheres, but the mantle is *not* a sphere! The mantle is the part of the Earth that is not core, so the volume of the mantle is:

Volume of mantle = (Volume of Earth) – (Volume of core) or $V_{mantle} = V_{Earth} - V_{core}$



Radius of Earth = 6370 km

Radius of core = 3486 km

Volume of Earth: ___1.08×10^{12}___km^3

Volume of Earth: ___1.08×10^{27}___cm^3

Volume of core: ___1.77×10^{11}___km^3

Volume of core: ___1.77×10^{26}___km^3

Volume of mantle: ___9.03×10^{11}___km^3

Volume of mantle: ___9.03×10^{26}___**cm^3**

HINTS: If you do the calculation using radii expressed in km, the answer you get will be in km^3 (cubic kilometers). You have been asked to express the answer in cm^3 (cubic centimeters). ***1 km^3 = 10^{15} cm^3*** (see part d for the derivation). Start by calculating the volume of the Earth and the volume of the core, then subtract to get the volume of the mantle. I think it is easiest to convert from km^3 to cm^3 as the last step, but you can do it sooner in the calculations if you wish.

Ve = 1.08×10^{27} *cm*^{3} *←From question #8b*

1.08×10^{27} cm^3 x (1 km^3 / 10^{15} cm^3) = 1.08×10^{12} km^3

Vc = (4/3) x π x (3486 km)^3 = 1.77×10^{11} km^3

1.77×10^{11} km³ x (10^{15} cm³ / 1 km³) = $\boxed{1.77 \times 10^{26} \text{ cm}^3}$

Vm = Ve – Vc = 1.08×10^{12} km³ – 1.77×10^{11} km³
= 10.8×10^{11} km³ – 1.77×10^{11} km³ = $\boxed{9.03 \times 10^{11} \text{ km}^3}$

9.03×10^{11} km³x (10^{15} cm³ / 1 km³) = $\boxed{9.03 \times 10^{26} \text{ cm}^3}$

b. Now calculate the <u>mass</u> of the mantle, in grams. Assume the density of the mantle is 4.5 g/cm³ (about 4.5 times the density of water). Remember that:

Density = Mass ÷ Volume which can be rearranged to ***Mass = Density x Volume***

For our calculations, the units to use are: ***Mass (g) = Density (g/cm³) x Volume (cm³)***
(Notice that the units of cm³ cancel, leaving g.)

Mass of mantle: _____<u>***4.06×10^{27}***</u>_____g

Mass = Density x Volume

Mass of mantle = Density of mantle x Volume of mantle

= 4.5 g/cm³ x 9.03×10^{26} cm³ = $\boxed{4.06 \times 10^{27} \text{ g}}$

c. Next, assume that the mantle is made up of material from stony meteorites. Stony meteorites contain about 0.5% water (half a percent, by weight). Calculate the mass of <u>water</u> in the mantle when it was formed from the meteorites. Express your answer in grams. (Take the mass of the mantle from part b, and figure out how much of that mass would have been water, given the proportions listed here.)

Mass of water in early-Earth mantle: _____<u>***2.03×10^{25}***</u>_____g

0.5% = 0.005

Mw(eem) = 4.06×10^{27} g x 0.005 = $\boxed{2.03 \times 10^{25} \text{ g}}$

d. Now calculate the volume of the <u>oceans</u>, in cubic centimeters. To do this, take the surface area of the oceans to be 3.6×10^{8} km². Take an average depth to be 3.8 km. Calculate the volume using the formula:

Volume (cm³) = Area (cm²) x Depth (cm) or ***Volume (km³) = Area (km²) x Depth (km)***

You may also want to use: 1 km = 10^5 cm → 1km³ = (10^5 cm)³ = 10^{15} cm³

Volume of seawater in the oceans: _____<u>***1.37×10^{24}***</u>_____cm³

Vo = 3.6×10^{8} km² x 3.8 km = 1.37×10^{9} km³

1.37×10^{9} km³ x (10^{15} cm³ / 1 km³) = $\boxed{1.37 \times 10^{24} \text{ cm}^3}$

e. Next calculate the mass of <u>pure water</u> in the oceans, in grams. (The saltiness came later.) Assume that the density of seawater is 1.025 g/cm^3 and that seawater is 96.5% pure water (the rest is "salts", etc).

Mass of pure water in the oceans: ______***1.35×10^{24}***______g

Mass = Density x Volume

Mw = Dw x Vw

96.5% = 0.965

Mw(o) = (0.965 x 1.025 g/cm^3) x 1.37 x 10^{24} cm^3 = 1.35 x 10^{24} g

f. Finally, *compare* the mass of pure water in the oceans (part e) to the mass of water originally in the mantle (part c). Which is bigger? By how much? Could the water of the oceans have come entirely from outgassing of the mantle? Explain.

To calculate which is bigger, compare your answers from part e and part c of this question. To determine how many grams difference there is, subtract the two values. To calculate the percentage difference, use:

$$\textbf{\% difference} = \left(\frac{\text{difference between the values}}{\text{the larger value}}\right) \textbf{x 100\%}$$

or

$$\textbf{\% difference} = \left(\frac{\text{larger value - smaller value}}{\text{larger value}}\right) \textbf{x 100\%}$$

The mass of ______***mantle water***__________ is bigger.

It is bigger by ______________________ grams, which is a _____________ % difference.

Could the water of the oceans have come entirely from outgassing of the mantle? **Yes** or **No** . Explain.

Mw(eem) – Mw(o) = 2.03 x 10^{25} g – 1.35 x 10^{24} g

= 20.3 x 10^{24} g – 1.35 x 10^{24} g = 1.90 x 10^{25} g

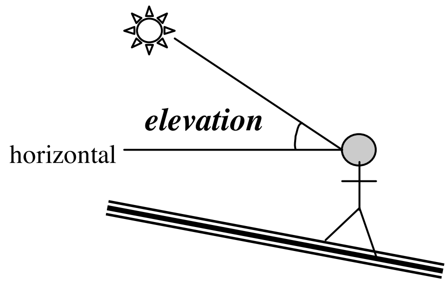
So: Yes, the amount of water in the early-Earth mantle was larger than the amount of water currently in the oceans.

% difference = (1.90 x 10^{25} g) / (2.03 x 10^{25} g) x 100% = (1.90/2.03) x 100% = 93.6%

Alternatively: Mw(eem) / Mw(o) = (2.03 x 10^{25} g) / (1.35 x 10^{24} g) = 15
That is, the mass of water originally in mantle was 15x greater than the mass of pure water now in the oceans. The mantle contained enough water to form the oceans. (This does not speak to other lines of evidence that some of the water may have come from comets/space.)

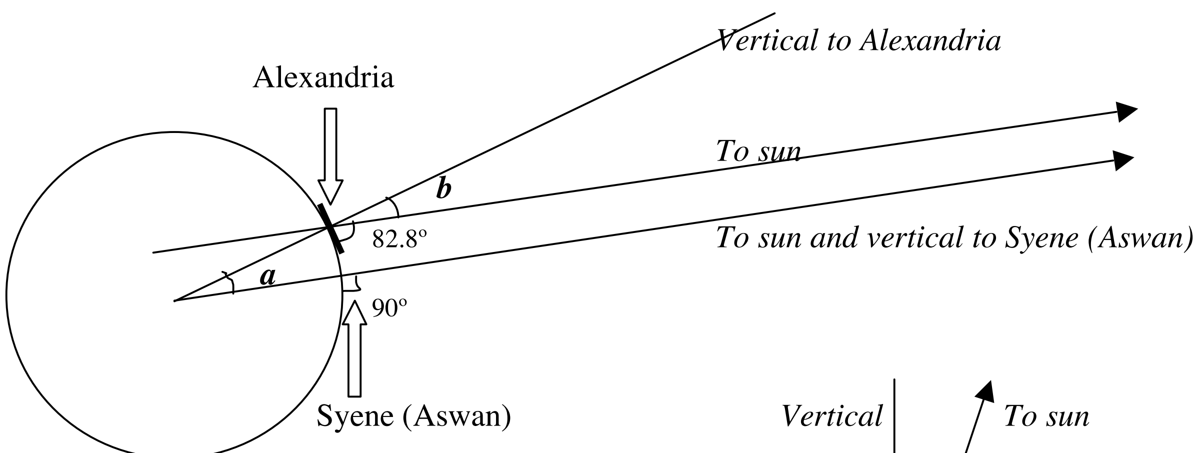
10. **<u>Here we will work through Eratosthenes' calculation of Earth's circumference (~230 B.C.).</u>**

The ***angle*** between the horizontal, an observer, and a celestial body is called the <u>elevation</u> of the body:

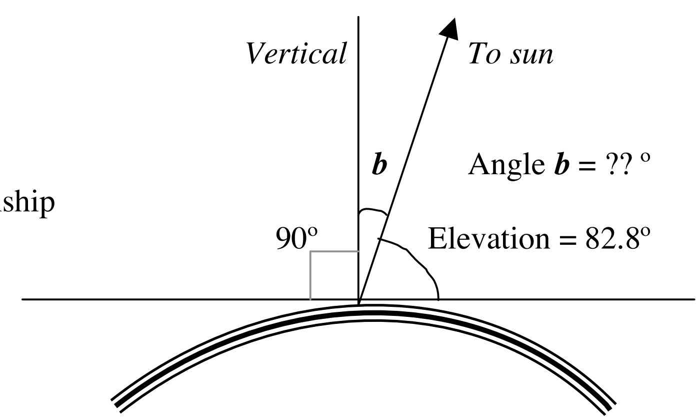


By measuring the elevation of the sun at two points along a north-south line on the surface of the Earth, and measuring the distance between those two points, it is possible to calculate the circumference of the Earth. Credit for the first measurement of this sort is often given to Eratosthenes (275-195 B.C.), a Greek who was the librarian of the famous library in Alexandria, Egypt. Eratosthenes noted that on one day each year the sun shone straight down a deep well in the city of Syene (now called Aswan). (It happened to be the summer solstice, but that is not essential to this measurement.) In other words, the elevation of the sun at noon in Syene on that day was 90º. At the <u>same time</u> in Alexandria, which is approximately 800 kilometers to the north of Syene, the elevation of the sun was 82.8º. (Noon is specified because at that time, shadows are aligned in a north-south direction.)

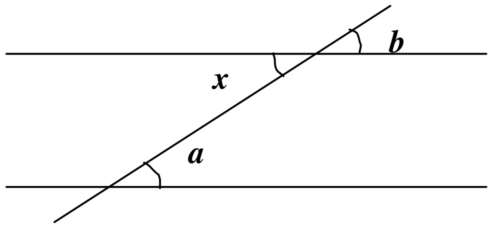
Below: On the day Eratosthenes made his measurements, at noon, the sun was directly overhead at Syene and was at an elevation of 82.8º at Alexandria. Because the sun is very far away from the Earth, the lines drawn to the sun from both Alexandria and Syene are parallel to each other.



At right: Close-up view of the relationship between the vertical and the direction to the sun at Alexandria.



As shown in the figure below, angle "***a***" (that is, the angle made by lines connecting the center of the Earth with Syene and Alexandria) is equal to angle "***b***" (that is, the angle made by a vertical line and a line pointing to the sun at Alexandria.)



angle ***a*** = angle ***x*** = angle ***b***

Above: When two parallel straight lines are cut by a third straight line, the interior angles formed are equal (that is, of the same size).

Using the preceding figures as a guide, and remembering that there are 360º in a circle (and 90º in a right angle), answer the following questions. ***Show all steps of your calculations.***:

a. How many degrees is angle "***a***", drawn from Alexandria to the center of the Earth to Syene? Explain your reasoning. (Hint: Be sure you look at all the diagrams!)

a = b = 90° – 82.8° = 7.2°

b. What fraction of 360º is the angle you just calculated?

F = 7.2° / 360° = 0.02 = 2% = 2/100

c. Given that the distance from Alexandria to Syene is 800 km, use the answer above to compute the circumference of the Earth. Give your answer in kilometers. ***Voila!***

800 km = 0.02 x Circumference

Circumference = 800 km / 0.02 = 40,000 km

Compare to:
http://geography.about.com/library/faq/blqzcircumference.htm
** The circumference of the earth at the equator is 24,901.55 miles (40,075.16 kilometers).*
** But, if you measure the earth through the poles the circumference is a bit shorter - 24,859.82 miles (40,008 km).*

d. How close is your calculation of the circumference of the Earth to the actual circumference? Explain some of the reasons why the two numbers are different (list as many possible reasons as you can).

Wow, that is very close! Good job, Eratosthenes!

(I'll let you think of some of the reasons the numbers are not exactly the same.)