11.0 MOVING BEYOND TWO DIMENSIONS

So far our study of calculus has taken place almost exclusively in the xy–plane, a 2–dimensional space. The functions we worked with typically had the form $y = f(x)$ so the graphs of these functions could be drawn in the xy–plane. And we have considered limits, derivatives, integrals, and their applications in two dimensions. However, we live in a three (or more) dimensional space, and some ideas and applications require that we move beyond two dimensions.

This chapter marks the start of our move into three dimensions and the mathematics of higher dimensions. The next several chapters extend the ideas and techniques of limits, derivatives, rates of change, maximums and minimums, and integrals beyond two dimensions. The work you have already done in two dimensions is an absolutely vital foundation for these extensions. As we work beyond two dimensions you should be alert for the the parts of the ideas and techniques that extend very easily (many of them) and those that require more extensive changes.

Section 11.1 introduces vectors and some of the vocabulary, techniques and applications of vectors in the plane. This section still takes place in two dimensions, but the ideas are important for our move into higher dimensional spaces.

Section 11.2 introduces the three–dimensional rectangular coordinate system, visualization in three dimensions, and measuring distances between points in three dimensions.

Section 11.3 extends the basic vector ideas, techniques and applications to three dimensions.

Sections 11.4 and 11.5 introduce two important types of multiplication, the dot product and the cross product, for vectors in 3–dimensional space and examines what they measure and some of their applications.

Section 11.6 considers the simplest objects, lines and planes, in 3–dimensional space.

Section 11.7 introduces surfaces described by second–degree equations and catalogs the possible shapes they can have.

Chapter 11 is the first step in our move beyond two dimensions. It contains the fundamental geometry of points and vectors in three dimensions. The concepts and techniques of this chapter are important and useful by themselves, and they are a necessary foundation for the study of calculus in three and more dimensions in Chapter 12 and beyond.