Section P.4  Solving Equations Algebraically and Graphically

Objective: In this lesson you learned how to solve linear equations, quadratic equations, polynomial equations, equations involving radicals, equations involving fractions, and equations involving absolute values.

Important Vocabulary

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
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<tbody>
<tr>
<td>Equation</td>
<td>A statement that two algebraic expressions are equal.</td>
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<tr>
<td>Extraneous</td>
<td>A solution that does not satisfy the original equation.</td>
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<tr>
<td>x-intercept</td>
<td>The point ((a, 0)) is an (x)-intercept of the graph if it is a solution point of the equation.</td>
</tr>
<tr>
<td>y-intercept</td>
<td>The point ((0, b)) is a (y)-intercept of the graph if it is a solution point of the equation.</td>
</tr>
<tr>
<td>Point of intersection</td>
<td>An ordered pair that is a solution of two different equations.</td>
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I. Equations and Solutions of Equations  (Pages 38–39)

To solve an equation in \(x\) means to . . . find all the values of \(x\) for which the solution is true.

The values of \(x\) for which the equation is true are called its solutions.

An identity equation is . . . an equation that is true for every real number in the domain of the variable.

A conditional equation is . . . an equation that is true for just some (or even none) of the real numbers in the domain of the variable.

A linear equation in one variable \(x\) is an equation that can be written in the standard form \(ax + b = 0\), where \(a\) and \(b\) are real numbers with \(a \neq 0\).

Example 1: Solve \(5(x + 3) = 35\).

The solution is 4.

To solve an equation involving fractional expressions, . . . find the least common denominator of all terms in the equation and multiply every term by this LCD.
When is it possible to introduce an extraneous solution?
When multiplying or dividing an equation by a variable expression.

Example 2: Solve: 
(a) \( \frac{5x}{7} = \frac{9}{14} \)  
(b) \( \frac{1}{x+1} + \frac{5x}{x^2-1} = \frac{4}{x-1} \)

(a) 0.9  
(b) 2.5

II. Intercepts and Solutions  (Pages 39–41)

To find the \( x \)-intercepts of the graph of an equation, . . . let \( y = 0 \) and solve the equation for \( x \).
To find the \( y \)-intercepts of the graph of an equation, . . . let \( x = 0 \) and solve the equation for \( y \).

Example 3: For the equation \( 12x - 4y = 12 \), find:
(a) the \( x \)-intercept(s), and (b) the \( y \)-intercept(s).
(a) (4, 0)  
(b) (0, -3)

III. Finding Solutions Graphically  (Pages 41–42)

To use a graphing utility to graphically approximate the solutions of an equation , . . . (1) write the equation in general form, \( y = 0 \), with the nonzero terms on one side of the equation and zero on the other side; (2) use a graphing utility to graph the equation. Be sure the viewing window shows all the relevant features of the graph; (3) use the zero or root feature or the zoom and trace features of the graphing utility to approximate each of the \( x \)-intercepts of the graph.

Example 4: Use a graphing utility to approximate the solutions of \( 3x^2 - 14x = -8 \).  
The solutions are \( 2/3 \) and 4.

IV. Points of Intersection of Two Graphs  (Pages 43–44)

To find the points of intersection of the graphs of two equations algebraically, . . . solve each equation for \( y \) (or \( x \)) and set the two results equal to each other. The resulting equation will be an equation in one variable, which can be solved using standard procedures.
To find the points of intersection of the graphs of two equations with a graphing utility, . . . use the graphing utility to graph both equations in the same viewing window and use the intersect feature or the zoom and trace features to find the point or points at which the two graphs intersect.

Example 5: Use (a) an algebraic approach and (b) a graphical approach to finding the points of intersection of the graphs of \( y = 2x^2 - 5x + 6 \) and \( x - y = -6 \).

\((0, 6)\) and \((3, 9)\)

V. Solving Polynomial Equations Algebraically (Pages 45–46)

List four methods for solving quadratic equations:
1) Factoring
2) Using the Square Root Principle or Extracting Square Roots
3) Completing the Square
4) Quadratic Formula

To solve a quadratic equation by factoring, . . . write the equation in general form with all terms collected on the left side and zero on the right. Then factor the left side of the quadratic equation as the product of two linear factors. Finally, find the solutions of the quadratic equation by setting each factor equal to zero.

Example 6: Solve \( x^2 - 12x = -27 \) by factoring.

The solutions are 3 and 9.

Using the Quadratic Formula to solve the quadratic equation written in general form as \( ax^2 + bx + c = 0 \) gives the solutions:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Example 7: For the quadratic equation \( 3x - 16 = -2x^2 \), find the values of \( a, b, \) and \( c \) to be substituted into the Quadratic Formula. Then find the solutions of the equation. Round to two decimal places.

\( a = 2, b = 3, \) and \( c = -16, \) OR \( a = -2, b = -3, \) and \( c = 16 \)

The solutions are \(-3.68\) and \(2.18\).
Example 8: Describe a strategy for solving the polynomial equation \( x^3 + 2x^2 - x = 2 \). Then find the solutions. First write the polynomial equation in general form with zero on the right-hand side of the equation. Then factor the polynomial by grouping to solve. The solutions are \(-2, -1,\) and \(1\).

VI. Other Types of Equations (Pages 47–49)

An equation involving a radical expression can often be cleared of radicals by . . . raising both sides of the equation to an appropriate power. When using this procedure, remember to check for ______ extraneous solutions, which do not satisfy the original equation.

Example 9: Describe a strategy for solving the following equation involving a radical expression:
\[
\sqrt{8-x} - 15 = 0
\]
Add 15 to both sides to isolate the radical expression. Then square both sides to eliminate the radical. Finally, solve for \(x\) and check the solution in the original equation.

To solve an equation involving fractions, . . . multiply both sides of the equation by the least common denominator of each term in the equation to clear the equation of fractions.

Example 10: Solve:
\[
\frac{2}{x} - 1 = \frac{1}{x + 1}
\]
\[\pm \sqrt{2}\]

To solve an equation involving an absolute value, . . . remember that the expression inside the absolute value symbols can be positive or negative, resulting in two separate equations to be solved.

Example 11: Write the two equations that must be solved to solve the absolute value equation \( |3x^2 + 2x| - 5 = 0 \).
\[
3x^2 + 2x = 5 \text{ and } -(3x^2 + 2x) = 5
\]