Section P.3 Lines in the Plane

Objective: In this lesson you learned how to find and use the slope of a line to write and graph linear equations.

Important Vocabulary

Slope The number of units a line rises or falls vertically for each unit of horizontal change from left to right.
Parallel Two distinct nonvertical lines are parallel if and only if their slopes are equal.
Perpendicular Two distinct nonvertical lines are perpendicular if and only if their slopes are negative reciprocals of each other. That is, \( m_1 = -\frac{1}{m_2} \).

I. The Slope of a Line (Pages 25–26)

The formula for the slope of a line passing through the points \((x_1, y_1)\) and \((x_2, y_2)\) is 
\[
m = \frac{y_2 - y_1}{x_2 - x_1}.
\]

To find the slope of the line through the points \((-2, 5)\) and \((4, -3)\), . . . subtract \(-3\) from \(5\) and divide this result by the difference of \(-2\) and \(4\).

A line whose slope is positive rises from left to right.
A line whose slope is negative falls from left to right.
A line with zero slope is horizontal.
A line with undefined slope is vertical.

II. The Point-Slope Form of the Equation of a Line  
(Pages 27–28)

The point-slope form of the equation of a line is 
\[
y - y_1 = m(x - x_1).
\]

This form of equation is best used to find the equation of a line when . . . you know the slope of a line and you also know the coordinates of one point on the line.

The two-point form of the equation of a line is 
\[
y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).
\]
The two-point form of equation is best used to find the equation of a line when . . . you know the coordinates of two points on the line.

**Example 1:** Find an equation of the line having slope \(-2\) that passes through the point \((1, 5)\).

\[
y = -2x + 7
\]

The approximation method used to estimate a point between two given points is called **linear interpolation**. The approximation method used to estimate a point lying outside the given points is called **linear extrapolation**.

**III. Sketching Graphs of Lines** (Pages 29–30)

The **slope-intercept form** of the equation of a line is

\[
y = mx + b
\]

where \(m\) is the **slope** and the \(y\)-intercept is \((0, b)\).

**Example 2:** Determine the slope and \(y\)-intercept of the linear equation \(42 = -2x\).

The slope is 2 and the \(y\)-intercept is \((0, -4)\).

The equation of a **horizontal line** is \(y = b\). The slope of a horizontal line is **0**. The \(y\)-coordinate of every point on the graph of a horizontal line is \(b\).

The equation of a **vertical line** is \(x = a\). The slope of a vertical line is **undefined**. The \(x\)-coordinate of every point on the graph of a vertical line is \(a\).

The **general form** of the equation of a line is

\[
Ax + By + C = 0
\]

Every line has an equation that can be written in **general form**.

When a graphing utility is used to sketch a straight line, the graph of the line may not visually appear to have the slope indicated by its equation because . . . of the viewing window used for the graph.
In general, two graphs of the same equation can appear to be quite different depending on . . . the viewing window selected.

**Example 3:** Use a graphing utility to graph the linear equation $2x - y = 4$ using (a) a standard viewing window, and (b) a square window.

(a) ![Graph (a)](image)

(b) ![Graph (b)](image)

**IV. Parallel and Perpendicular Lines** (Pages 31–32)

Two lines are parallel if they do not intersect.
Two lines are perpendicular if they intersect at right angles.

The relationship between the slopes of two lines that are parallel is . . . that the slopes are equal.

The relationship between the slopes of two lines that are perpendicular is . . . that the slopes are negative reciprocals of each other.

A line that is parallel to a line whose slope is 2 has slope $2$.
A line that is perpendicular to a line whose slope is 2 has slope $-\frac{1}{2}$.

**Example 4:** Use a graphing utility to graph the perpendicular lines $y = 2x - 3$ and $y = -0.5x + 5$ using (a) a standard viewing window, and (b) a square window.

(a) ![Graph (a)](image)

(b) ![Graph (b)](image)
Additional notes