

## Section 3.4 Solving Exponential and Logarithmic Equations

Course Number

Instructor

Date

**Objective:** In this lesson you learned how to solve exponential and logarithmic equations.

### I. Introduction (Page 247)

State the One-to-One Property for exponential equations.

$$a^x = a^y \text{ if and only if } x = y$$

State the One-to-One Property for logarithmic equations.

$$\log_a x = \log_a y \text{ if and only if } x = y$$

State the Inverse Properties for exponential equations and for logarithmic equations.

$$a^{\log_a x} = x \quad \text{and} \quad \log_a a^x = x$$

Describe how the One-to-One Properties and the Inverse Properties can be used to solve exponential and logarithmic equations.

- Rewrite the given equation in a form that allows the use of the One-to-One Properties of exponential or logarithmic functions to solve the equation.
- Rewrite an exponential equation in logarithmic form and apply the Inverse Property of logarithmic functions to solve the equation.
- Rewrite a logarithmic equation in exponential form and apply the Inverse Property of exponential functions to solve the equation.

**Example 1:** (a) Solve  $\log_8 x = \frac{1}{3}$  for  $x$ .

(b) Solve  $5^x = 0.04$  for  $x$ .

(a)  $x = 2$     (b)  $x = -2$

### II. Solving Exponential Equations (Pages 248–249)

Describe how to solve the exponential equation  $10^x = 90$  algebraically.

Take the common logarithm of each side of the equation and then use the Inverse Property to obtain:  $x = \log 90$ . Then use a calculator to approximate the solution by evaluating  $\log 90 \approx 1.954$ .

#### *What you should learn*

How to solve simple exponential and logarithmic equations

#### *What you should learn*

How to solve more complicated exponential equations

**Example 2:** Solve  $e^{x-2} - 7 = 59$  for  $x$ . Round to three decimal places.

$$x \approx 6.190$$

### III. Solving Logarithmic Equations (Pages 250–252)

Describe how to solve the logarithmic equation

$$\log_6(4x - 7) = \log_6(8 - x) \text{ algebraically.}$$

Use the One-to-One Property for logarithms to write the arguments of each logarithm as equal:  $(4x - 7) = (8 - x)$ . Then solve this resulting linear equation by adding 7 to each side, adding  $x$  to each side, and then finally dividing both sides by 5. The solution is  $x = 3$ .

**Example 3:** Solve  $4 \ln 5x = 28$  for  $x$ . Round to three decimal places.

$$x \approx 219.327$$

### IV. Approximating Solutions (Page 252)

Describe at least two different methods that can be used to approximate the solutions of an exponential or logarithmic equation using a graphing utility.

Graph the left-hand side and the right-hand side of the equation in the same viewing window; then use the intersect feature or the zoom and trace features of the graphing utility to find the points of intersection. OR rewrite the equation so that all terms on the left side are equal to 0. Then use a graphing utility to graph the left side of the equation. Use the zero or root feature or the zoom and trace features to approximate the solutions of the equation.

### V. Applications of Solving Exponential and Logarithmic Equations (Page 253)

**Example 4:** Use the formula for continuous compounding,  $A = Pe^{rt}$ , to find how long it will take \$1500 to triple in value if it is invested at 12% interest, compounded continuously.

$$t \approx 9.155 \text{ years}$$

#### *What you should learn*

How to solve more complicated logarithmic equations

#### *What you should learn*

How to approximate the solutions of exponential or logarithmic equations with a graphing utility

#### *What you should learn*

How to use exponential and logarithmic equations to model and solve real-life problems

#### Homework Assignment

Page(s)

Exercises