Section 3.3 Properties of Logarithms

Objective: In this lesson you learned how to rewrite logarithmic functions with different bases and how to use properties of logarithms to evaluate, rewrite, expand, or condense logarithmic expressions.

I. Change of Base (Page 240)

Let \(a\), \(b\), and \(x\) be positive real numbers such that \(a \neq 1\) and \(b \neq 1\). The change-of-base formula states that . . .

\[ \log_a x \] can be converted to a different base using any of the following formulas:

Base \(b\): \[ \log_a x = \frac{(\log_b x)}{(\log_b a)} \]
Base 10: \[ \log_a x = \frac{(\log_{10} x)}{(\log_{10} a)} \]
Base \(e\): \[ \log_a x = \frac{(\ln x)}{(\ln a)} \]

Explain how to use a calculator to evaluate \(\log_8 20\).

Using the change-of-base formula, evaluate either \((\log 20) ÷ (\log 8)\) or \((\ln 20) ÷ (\ln 8)\). The results will be the same: 1.4406

II. Properties of Logarithms (Page 241)

Let \(a\) be a positive number such that \(a \neq 1\); let \(n\) be a real number; and let \(u\) and \(v\) be positive real numbers. Complete the following properties of logarithms:

1. \[ \log_a (uv) = \log_a u + \log_a v \]

2. \[ \log_a \frac{u}{v} = \log_a u - \log_a v \]

3. \[ \log_a u^n = n \log_a u \]

III. Rewriting Logarithmic Expressions (Page 242)

To expand a logarithmic expression means to . . . . . . use the properties of logarithms to rewrite the expression as a sum, difference, and/or constant multiple of logarithms.

Example 1: Expand the logarithmic expression \(\ln \frac{xy^4}{2}\).

\[ \ln x + 4 \ln y - \ln 2 \]
To condense a logarithmic expression means to . . . use the properties of logarithms to rewrite the expression as the logarithm of a single quantity.

**Example 2:** Condense the logarithmic expression

\[
3 \log x + 4 \log(x - 1) - \log[x^8(x - 1)^5]
\]

**IV. Applications of Properties of Logarithms** (Page 243)

One way of finding a model for a set of nonlinear data is to take the natural log of each of the \(x\)-values and \(y\)-values of the data set. If the points are graphed and fall on a straight line, then the \(x\)-values and the \(y\)-values are related by the equation:

\[
\ln y = m \ln x + c
\]

where \(m\) is the slope of the straight line.

**Example 3:** Find a natural logarithmic equation for the following data that expresses \(y\) as a function of \(x\).

\[
\begin{array}{c|cccc}
  x & 2.718 & 7.389 & 20.086 & 54.598 \\
  y & 7.389 & 54.598 & 403.429 & 2980.958 \\
\end{array}
\]

\[
\ln y = 2 \ln x \quad \text{or} \quad \ln y = \ln x^2
\]

**Homework Assignment**

Page(s)

Exercises