

## **Bellevue School District Differential Equations** (= Bellevue College Math 238)

Prerequisites: AP Calculus (1 year) and Advanced Calculus (1 year)

Because of the excellent students in the class and because we have the luxury of an entire year, this class covers both the “classical” analytic and the more modern “qualitative” approaches to introductory Differential Equations. The level is “university honors” but much slower paced. The class meets daily 7:00 – 7:55 AM.

**Part I** (about 5 months): "classical" ODE course with the focus on closed form, by-hand solutions.

### **Elementary Differential Equations by Boyce & DiPrima**

a "classic" approach of getting closed form solutions by hand

#### Introduction

classification of DEs

linear equations: integrating factors

separation of variables

modeling with first order ODE (mixing, falling bodies)

autonomous eqs. and population dynamics

assorted population models

exact eqs – recognizing and solving (+ graphic interpretation)

numerical methods -- Euler's method

#### Homogeneous second order with constant coefficients

linear independence and the Wronskian

characteristic equation and its roots

complex roots of the characteristic eq (and Euler's equation)

solutions to homogeneous eqs using the characteristic eq roots

nonhomogeneous: undetermined coefficients

nonhomogeneous: variation of parameters

mechanical vibrations – springs (and electrical vibrations)

    resonance, damping (under, over, critical), frequency,

    root loci (parameters  $\leftrightarrow$  roots  $\leftrightarrow$  system behavior)

    forcing, steady state, beats

#### Series solutions of ODEs

review of power series, Taylor series for  $\exp(x)$ ,  $\sin(x)$ ,  $\cos(x)$  and variations

second order linear with constant coefficients

solving 2nd order ODEs with variable coefs

    including the Airy functions as solutions and a Bessel function

Cauchy-Euler equations (put  $y = x^r$ )

#### Laplace transforms $L(f)$

definition of  $L(f)$

calculating  $L(f)$  from the definition (even for some piecewise continuous fncs)

inverse transforms and using a table of transforms

solving of initial value problems using  $L(f)$

step functions

DEs with discontinuous forcing functions  
impulse functions -- Dirac delta function

**Part II** (about 3 months): "modern" approach that emphasizes systems and the qualitative/visual understanding of solutions.

**Differential Equations by Blanchard, Devaney and Hall**

a "modern" geometric/qualitative approach

modeling via systems (all 2x2, standard model is competing species)

review of 2x2 matrix arithmetic

matrix representation of a 2x2 system

geometry of systems

some analytic methods

Euler's method for systems

a bit of Lorenz eqs (chaos)

Linear systems (all 2x2)

finding eigenvalues and eigenvectors of a system

properties and the linearity principle

real eigenvalues: straight line solutions in the phase plane

complex eigenvalues: converting to real "sin/cos" solutions, spiral solutions

what eigenvalues alone tell us about the phase plane and equilibrium points

source, sink, saddle, straight line solutions, spirals, stability

systems and 2nd order linear eqs

equilibrium point analysis

trace-determinant plane

linearization

classification of equilibrium points (source, sink, spiral, saddle) after linearization

Comparing 2x2 linear systems and 2nd order constant coef ODEs

eigenvalues and characteristic eq roots

Visiting lectures to end the year in 2009

Dr. Frenichel:  $y = \exp(At)$  solutions where  $A$  is a 2x2 matrix of special type: motivation, derivation, system solutions

Q. Yuan (now at MIT): 2x2 matrix generation of the Fibonacci sequence, generating equations, eigenvalues

K. Danesh (now at Duke): A linked 5x5 system to model proteins in human circadian rhythm and sleep cycle disorders