1. Fig. 1 shows the graphs of $r(t)$, $\theta(t)$, and $z(t)$ as functions of time $t$. Sketch the path of an object whose cylindrical coordinate location at time $t$ is $(r(t), \theta(t), z(t))$.

2. Describe in words and with a sketch, the surface we get in spherical coordinates for $(3, \theta, \phi)$ for $0 \leq \theta \leq 360^\circ$ and $0 \leq \phi \leq 30^\circ$.

3. $f(x,y) = \frac{x^2}{x^2 + 2y^2}$ (Show your work. No work = no points.)

   (3) (a) Calculate the limit of $f(x,y)$ as $(x,y)$ approaches $(3,-1)$: 

   (3) (a) Calculate the limit of $f(x,y)$ as $(x,y)$ approaches $(0,0)$: 

4. \[ f(x,y) = x^2 y^3 + 2x - 2y^3 + 5 \] (All questions on this page refer to this function.)

(4) (a) \[ f_x(x,y) = \] \hspace{1cm} (1) Evaluate \[ f_x(2,1) = \] 

(4) (b) \[ f_y(x,y) = \] \hspace{1cm} (1) Evaluate \[ f_y(2,1) = \] 

(4) (c) \[ f_{yx}(x,y) = \] 

(4) (d) At the point \( (2, 1) \), \[ dz = \] 

(4) (e) Use the previous result to estimate the value of \[ f(1.9, 1.3) \approx \] 

(5) (f) Write an equation for the plane tangent to the graph of \( f(x,y) \) at the point \( (2, 1, 11) \).

\[ z = \] 

(4) (g) \[ \nabla f(x,y) = \] \hspace{1cm} (2) \[ \nabla f(2,1) = \] 

(5) (h) When \( u = 0.6i - 0.8j \), \[ D_u f(2,1) = \] 

(4) (i) The maximum value of \( D_u f(2,1) \) is \[ \] 

and this maximum occurs when \( u = \{ \quad , \quad \} \) (fill blanks with decimal #s)
5. The surface area $S$ (in m$^2$) of a person with weight $w$ (in kg) and height $h$ (in cm) is

$$ S(w,h) = 0.007 w^{0.4} h^{0.7} $$

so a 50 kg person who is 160 cm tall has a surface area of

$$ S(50, 160) = 1.17 \text{ m}^2 \text{ (approximately)}. $$

(a) What are the units of $S_w$? ________________ Units of $S_h$? ________________

(b) Calculate $S_w(w, h)$ at the point $(50, 160) = \_____________ \text{(include units).}$

(c) Explain the meaning of $S_w(50, 160)$ in part (a) as it applies to the 50 kg, 160 cm person.

6. The temperature at the location $(x, y, z)$ in a room is $T(x,y,z) = x^2 + 2y^3 + xz$ degrees. An object is moving along the path given parametrically by $x = 1 + 2w$, $y = \sin(3w)$, $z = 5 + 3w + w^2$ at time $w$

(a) Draw the map (web, tree) showing how the variables are connected.

(b) Use the Chain Rule to write the formula for

$$ \frac{dT}{dw} = $$

(c) Calculate $\frac{dT}{dw}$ when $w = 0$: $\frac{dT}{dw} = \_____________ \text{ (number)}$
7. Several depth readings for a portion of a lake are shown in the figure.
Sketch "reasonable" level curve(s) for the depth 4.

8. The diagram shows some elevation level curves of the function \( z = g(x,y) \).

(a) At C, \( g_x \) is + – 0 (circle one)
(b) At C, \( g_y \) is + – 0 (circle one)

(c) On the axis system below, sketch the graph of your elevation as you move along the dark curve from point A to point B.

BONUS (+3 if correct) What is a "Mobius Strip" and what is its remarkable property?

The end!! Tests back tomorrow. Important class -- last class before AP Break -- be here!!