Math 126

Show Your Work! October 14, 2003 Name _____
Good Luck! Test #1 (please print)

1. Fig. 1 shows the values of r as a function of θ for $0 \le \theta \le 2\pi$. Sketch the polar coordinate graph of (r, θ).

(5)

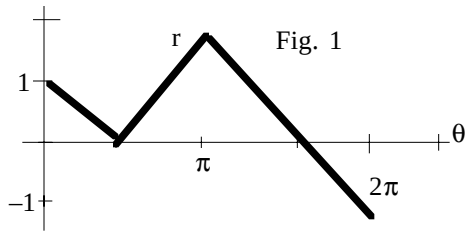
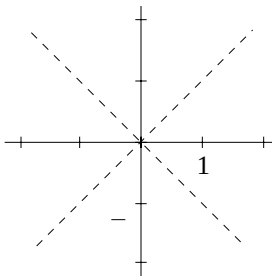


Fig. 1



2.. The graph of $r = 1 + \sin(\theta)$ is shown.

(a) Represent the area of this cardiod as a definite integral.
(DO NOT EVALUATE.)

(4)

one petal area = $\int$

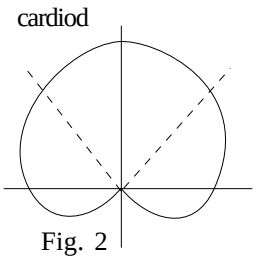


Fig. 2

(2) (b) Plot the (x,y) location of (r, θ) when θ = 1.4 (radian, of course)?

x = _____ = _____

(c) Find the following when θ = 1.4
(Show your work and give 2 decimal places in your answers.)

(3) $\frac{d\,r}{d\,\theta}$ = _____

(4) $\frac{d\,x}{d\,\theta}$ = _____

3. Give the equations of the asymptotes The graph of $\frac{y^2}{25} \pm \frac{x^2}{49} = 1$ is a hyperbola.

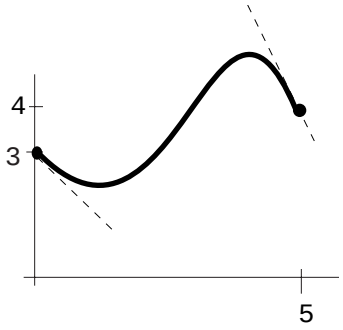
(1) (a) Give the coordinates of one point on the hyperbola: (_____ , _____)

(4) (b) What are its asymptotes? y = _____ and y = _____

4. Which conic section is the graph of $r = \frac{7}{3 + 5\cos(\theta)}$? _____

(3)

5. Find reasonable parametric equations ot describe a curve that goes through the point (0, 3) with slope –1 and through the point (5, 4) with slope –2 (see Figure 4). (think Bezier! and **do not simplify** the equations)



List your control points: ( ), ( ), ( ), ( )

(8) x(t) = _____

y(t) = _____

6. Biographies -- short fill–ins. (One point each.)

(a) _____

(b) _____

(c) _____

7. The location of the point P at time t seconds is
x(t) = $x(t) = t^3 \pm 4t^2 + 4t$ feet and
$y((t) = t + 1$ feet.

graph

(a) **Graph** the path of the point for $0 \le t \le 3$ and **label** the coordinates of the maximum values for x and y.

(4)

(b) Find $\frac{d\,y}{d\,x}$ when t = 2: $\frac{d\,y}{d\,x}$ = _____ (give 2 decimal places)

(6)

(c) Find the **speed** of the point when t = 2: speed = _____ (give 2 decimal places)

(4)

(d) Represent the **length** of the path as a definite integral for $0 \le t \le 3$. (DO NOT EVALUATE.)

(4)

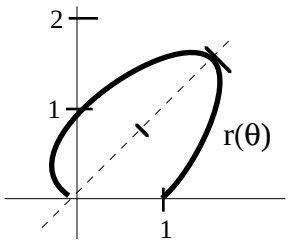
Length = $\int$

(e) Represent the **area** enclosed between the path of the point and the x–axis (for $1 \le t \le 2$) as a definite integral. (DO NOT EVALUATE.)

(4)

Area = $\int$

8. The polar coordinate graph of $r = r(\theta)$ is shown.



(a) Sketch a rectangular coordinate graph of

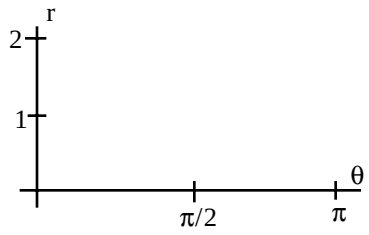
(4) (θ, r) for $0 \le \theta \le \pi$

(b) When $\theta = \pi/3$ (not $\pi/4$)

(6) $\frac{dr}{d\theta}$ is + – 0 undefined

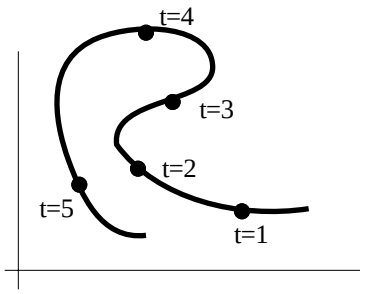
$\frac{dx}{d\theta}$ is + – 0 undefined

$\frac{dy}{dx}$ is + – 0 undefined



9. The parametric equation graph (x(t), y(t)) is shown.

When $t = 2$,



(6) $\frac{dx}{dt}$ is + – 0 undefined

$\frac{dy}{dt}$ is + – 0 undefined

$\frac{dy}{dx}$ is + – 0 undefined

10. Explain, **in words and pictures**, the reflection property of the **parabola**.

(Use good English and complete sentences.)

(4)

11. (a) $a_1 = 3$, $a_2 = 1$, and $a_n = (a_{n-1})^2 + a_{n-2}$. Calculate a_3 = _____ and a_4 = _____

(2)(2)

For parts (b) – (e), give exact answers or answers correct to two decimal places.

(b) $\lim_{n\to\infty} \frac{\ln(n)}{2n}$ = _____

(3)

(c) $\lim_{n\to\infty} \left(1 - \frac{2}{n}\right)^n$ = _____

(3)

(d) $\lim_{n\to\infty}$ arctan(3n) = _____

(3)

(e) $\lim_{n\to\infty}$ $1 + 0.5 + (0.5)^2 + ... + (0.5)^n$ = _____

(3)

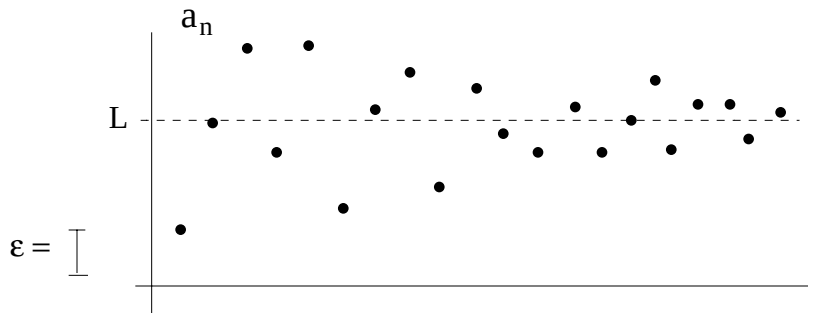
12. $\lim_{n\to\infty} \left(5 + \frac{7}{n^3}\right) = 5$. Find the best (smallest) value of N so we can be certain that

$\left|\left(5 + \frac{7}{n^3}\right) \pm 5\right| \leq 0.007$ whenever $n \geq N$. (Show your work.) N = _____

(3)

13. The figure shows a sequence a_n with $\lim_{n\to\infty} a_n = L$. A value for epsilon (ε) is shown graphically. At the appropriate locations on the graph, plot and label the points L+ ε, L– ε , and the value N that satisfies the definition of limit of a sequence for this sequence and ε.

(6)



the end!!